

First-year Seminar: Mathematics Beyond Calculus

Math 199i, Spring 2019

1. TECHNICAL INFORMATION

Instructor: Josh Sabloff (jsabloff@haverford.edu, KINSC H213)

Office Hours: See Moodle for updated office hours

Pre- / co-requisite: Math 215 (Linear Algebra)

2. GOALS OF THE COURSE

This half-course aims to lead you to broaden your perspective on what it means to do mathematics. You will encounter mathematics that usually lies beyond the introductory calculus and linear algebra sequence. You will gain familiarity with the formulation of a mathematical problem, develop strategies for investigating problems, and explore the mathematical literature. In addition, you will improve your ability to think rigorously and communicate mathematical information.

You will apply these modes of thought to the study of mathematical knots. A mathematical knot is like the knots you use to tie your shoes, with two differences: first, a mathematical knot has no ends, that is, it is a knotted-up circle. Second, a mathematical knot has no thickness. Two knots are defined to be equivalent if one can be deformed into the other without cutting the string. A natural question — at least for a mathematician — is whether there exists a general and effective method for determining if two knots are, or are not, equivalent.

You will explore this question (and more) by using knot diagrams, exploring interesting families of knots, and defining “invariants” of knots. An invariant is an object you assign to a knot that does not change when the knot is deformed. That object can be as simple as an integer or a polynomial, though they can get much more complicated. If two knots have different invariants, then it is impossible to deform one into the other.

3. CLASS TIME

Class time will be a mix of short, interactive lectures to introduce new ideas; discussion of the topics at hand and perhaps problems on the prior week’s assignment; and guided collaborative problem-solving and exploration.

At the end of every class meeting, we will agree upon one or more directions to explore for next time. I will post an assignment based on these directions by Thursday at noon.

4. ASSIGNMENTS

4.1. Weekly Assignments. For the weekly class meeting, you will be expected prepare as follows:

- (1) Explore one of the directions we have agreed to discuss next time.¹ Do examples. Make conjectures. Try to prove them. Fail. Modify the conjecture. Do more examples. Try again. If I have given permission, look at the mathematical literature (otherwise, please do not consult any outside sources). Collaboration is welcome.

Be prepared to discuss your explorations, even bringing up partial results (“This conjecture works for some examples”) or negative results (“This idea seemed promising, but did not work”).

- (2) Write up solutions to the posted exercises. There will be around 2 exercises designed to solidify the previous week’s discussion. You should hand in your solutions on Moodle by Monday at 11:59pm. The exercises will be graded on a 0–3 point scale. Collaboration is welcome, but you must write up your solutions on your own, out of sight of any notes you took during your collaborations.

NOTE: Following the rule of thumb that you should spend two hours outside of class for every one hour in class, I expect an average of three hours of outside work from you each week, with somewhat less at the beginning and a little more around the time the mini-projects are due.

4.2. Mini-Projects. To complement the weekly assignments, you will complete two mini-projects that allow you to delve more deeply into an interesting class of examples or a piece of theory that we did not have time to cover in class.

4.2.1. *Written Report.* Your 2–4 page written report should completely and clearly answer the question proposed, give context for the material, and be aimed at an audience of your peers. Some (though not all) of the mini-projects will require you to do some research in textbooks or in the primary literature; in this case, careful citation is essential. See the “What is a Mini-Project?” document for a more detailed description.

You will hand in a draft of each mini-project report one week before the final due dates.

4.2.2. *Oral Presentation.* For one of your mini-projects, you will give a 10-minute oral presentation of your report. The time constraint means that you cannot say everything, and certainly cannot give all details! What is the most important idea? What techniques did you use? How does your mini-project relate to other material from class?

You are more than welcome to discuss your presentation with me beforehand.

¹If you would prefer to work on a different question, please tell me and we can negotiate. I promise that you will “win” the negotiation almost every time, at least at the level of me telling you to work on your question for the mini-project.

5. GRADING

Your grade for the course is determined by:

Weekly Assignments: 20%

Written Mini-Projects: 35% each

Oral Presentation: 10%

I will also take your trajectory into account, as I expect there to be some growing pains at the beginning of the class.

6. RESOURCES

Haverford College is committed to providing equal access to students with a disability. If you have (or think you have) a learning difference or disability including mental health, medical, or physical impairment, please contact the Office of Access and Disability Services (ADS) at hc-ads@haverford.edu. The Coordinator will confidentially discuss the process to establish reasonable accommodations.

Students who have already been approved to receive academic accommodations and want to use their accommodations in this course should share their verification letter with me and also make arrangements to meet with me as soon as possible to discuss their the specific accommodations. Please note that accommodations are **not retroactive** and require advance notice to implement.

It is a state law in Pennsylvania that individuals must be given advance notice if they are to be recorded. Therefore, any student who has a disability-related need to audio record this class must first be approved for this accommodation from the Coordinator of Access and Disability Services and then must speak with me. Other class members will need to be aware that this class may be recorded.