Math 335: Topology

Fall 2019

1. Technical Information

Instructor: Josh Sabloff (`jsabloff@haverford.edu`)

Office Hours: M 1:30–3:30, Tu 12–1 and 2:30–3:30, W 2–3, F 1:30–2:30. These hours are subject to change. Please see Moodle for the most up-to-date schedule.

Texts: There are no texts for this course. In fact, for reasons I will explain below, I will view the use of any textbook or other outside resource in this course as a violation of the Honor Code, with the exception of your analysis text and, in the last part of the course, your algebra text.

2. Goals of the Course

2.1. **Content.** The goal of this course is to introduce you to the fundamental examples, problems, and machinery of topology. The central concepts in topology are those of *nearness* of points in a set and *continuity* of functions between sets for which a concept of nearness has been defined. The goal of topology is then to study properties of sets that are preserved under continuous functions, as well as properties of the continuous functions themselves. The result is a general and powerful theory that can make deep qualitative statements in a variety of situations, from the classic "rubber sheet geometry" formulation of topology, to vector calculus, dynamical systems, and even to robot motion planning or data analysis!

2.2. **Growth in Mathematical Thinking.** This course is structured to immerse you in the creative process of making new (to you) mathematics. You will prove theorems, construct (counter-) examples, and perhaps make conjectures. You will hone your mathematical thought processes in writing, speaking, and critiquing the work of others. You will gain increased ownership of and accountability for the material and your own mathematical processes.

Please be aware that this is a less linear (and more frustrating) growth process than listening to lectures, reading a textbook, and doing problem sets — but it is also more rewarding. There is a very prevalent belief that you are either "good" or "bad" at math, and if you are "bad" at it, then you will always be bad at it no matter how hard you try. This is extremely false,[1] and the mathematics community bears a lot of responsibility for perpetuating this myth. In reality, mathematics is just like any other discipline or skill: you can improve more and more with practice, and this class is designed to challenge your practice. You should measure your success in this class by how much your understanding of the concepts have improved over the course of the semester.

[1] I promised myself that I would not swear in the syllabus.

3. Course Structure

3.1. **The Moore Method.** This course will be taught using a version of the Moore Method (a.k.a. inquiry-based learning). This means that instead of the more standard lecture-homework-exam course structure, I will give you course notes that consist of definitions, theorems, examples, and exercises (though yes, there will still be a final). *You* and your classmates will prove the theorems and work out the examples, both in writing and in class, with some guidance from me and structured collaboration with your classmates.

We will *all* work to make the classroom a welcoming community for all in which to experiment, collaborate, and learn. In particular, remember that we are not all coming to this class with the same privileges, resources, time, and knowledge. As a community, mathematicians and scientists need to do a much better job of making our disciplines more accessible to people of all races, genders (including gender non-conforming folks), sexual identities, and class backgrounds. While this is a priority for us in the classroom, we do not claim to know how to best honor this commitment, and so we are very open to feedback from you when it comes to making the course more accessible and inclusive to all identities.

I highly suggest reading the advice of former Moore Method students, which is available on Moodle.

I hope that you now understand the reason behind the following edict:

> *You are not allowed to use* any *outside sources — textbooks, the internet, notes from students who took the course before, etc. — in this class, with the exception of your analysis and algebra textbooks. I will consider any such use to be a violation of the Honor Code.*

3.2. **In-Class Presentations.** In-class presentations are the lifeblood of this course.

A typical day in class will look like the following: the evening before the class, students will email me,[2] volunteering to present one or more propositions from the class notes that I had suggested in the previous class. *Not all propositions in the class notes will be presented in class* — some will only appear on homework, and some will simply be passed over unless you are interested in proving them on your own. You may volunteer a partial proof, or a proof of a special case, if that is where you are in the process. I will tell you if you have been chosen to present by about 8am. Please trust that I have my reasons for selecting who presents what.

At the beginning of the class, I will ask the volunteers to present proofs (or partial proofs) of propositions from the class notes. On a normal day, several students will present proofs to the class; I might also get up to make a brief presentation if the material warrants it. You may take notes to the board with you. If — *when* — you make a mistake, don't panic. You will be given a chance to correct it on the fly, and if you cannot, I will either ask you to try again during the next class session or will let another student present, depending on the nature of the mistake and your preference.

[2]You may send this email as late at night as you want, so long as I get it when I wake up in the morning. But be careful: I have young children, so I wake up quite early.

If you are not presenting, then you should not relax: it is your responsibility to help the class (politely and respectfully) ensure that the presenter's proof is correct. You may ask questions of the presenter, but you may not give away answers and you may *certainly* not take over their presentation. You and the rest of the class are in this venture together — without everyone both presenting and critiquing at a high level, we will not get any math done!

After each presentation, we (you) will discuss the presenter's argument to clarify or correct specific points, offering praise where warranted and constructive criticism when necessary. The day's Host will facilitate this discussion. We may then proceed to analyze the statement and proof further, by, for instance, examining the effect of changing hypotheses, discussing whether a converse holds, or thinking about whether the proof is good enough to prove a stronger statement. Conjectures are most welcome at this point! At the end of class, I will suggest next steps in the notes or perhaps inform you that I will put up a new section of the notes.

Presentations will be graded for the correctness of the mathematics (2 points), the (logical) structure of the presentation (2 points), and mastery of the technicalities (one point). We will discuss exactly what goes into each of these facets on the first day. *The first presentation you make will only be graded for completeness. Further, there is no penalty for a presentation that you have to abandon because of a serious mathematical error; such a presentation will simply go unrecorded.*

You need to present at least four times during the semester to earn a passing grade. Do not panic: there are plenty of theorems or examples in the course notes, so there are plenty of presentations to go around.

3.3. **Written Work.** You may hand in proofs of propositions in the class notes in five different ways:

(1) You may turn in, at any time, a written proof of a proposition that has yet to be presented in class or assigned for homework and is in the current subsection of the notes. Please tag these proofs as "new" in your writeup.
(2) If you present a problem, you must turn in a written proof within 24 hours of the presentation. Please tag these proofs as "new" in your writeup. Please alert me when you upload such a proof so I can include it in the notes as a "draft" after your presentation.
(3) You may turn in, at any time, a written proof of a proposition that *has* been presented in class and is in the current subsection of the notes. These proofs should *not* be labeled "new" in your writeup.
(4) I will assign write-ups of proofs that have already been presented and/or a few new exercises, depending on the week. These will be due every Monday by 5pm. I will inform you of what you need to turn in by the end of the prior Wednesday. While you are welcome to take notes in class, you may not look at those notes while writing up your proofs (see the "Collaboration Guidelines" section below). If you are handing in enough new proofs, or at least working ahead, you will probably never have to do a homework assignment from scratch. These proofs should *not* be labeled "new" in your writeup.

(5) You may rewrite proofs that I have graded *for full credit* at any time and as many times as you would like until I "close" a section of the notes (typically about two weeks after we are done presenting proofs from that section).

The grading rubric for the written work is as follows. For proofs whose first version is handed in by the original due date:

4 points: The proof is correct and clearly written, which almost always means that the author has explained not only the steps of the proof, but the strategy of the proof.
3 points: The proof suffers from a minor defect in correctness or clarity, though the author seems to understand the proof, and the author has turned in *at least two* drafts.
0 points: All other proofs. This grade will appear as a "RW" (meaning "rewrite") on your paper (really, "paper" — see below).
+1 point: Proofs submitted using the first criterion, above, so long as that proof is deserving of a 3 or 4 the first time around.
+2 points: Proofs of particularly difficult statements, as designated in the course notes, so long as the proof eventually deserves and 3 or 4.

Late work is subject to a 50% penalty.[3] Note that the absolute maximum score on a typical proof is 5, though *expected* maximum score is 4.

Some technicalities:

- You must type your written work in LaTeX. I will provide sample and template files for you to use. If you have never before used LaTeX, please see `http://www.haverford.edu/mathematics/resources/LaTeX.php` for instructions; you may also talk to me or contact David Lippel.
- You must write in complete sentences and complete paragraphs; as tempting as it is, you should mostly avoid using technical symbols like $\exists$, $\forall$, and $\Rightarrow$, as they make your prose all but unreadable except in rare circumstances. Remember: you are writing the textbook for this class!
- You should "hand in" your written work by placing a PDF file in the "New Submissions" subfolder of the Dropbox folder that you and I will be sharing *and* sending me an email when you have done so. Please use the following file naming convention:

  `your-last-name_section-subsection_date.pdf`

  For example, if your name is Judy Smith and you are handing in problems from Section 4.3 on September 15th, then the name of your file should be:

  `smith_4-3_9-15.pdf`

  In particular, if you are handing in proofs from different sections, they should go in different files.

[3] I am, of course, willing to stretch the definition of "on time" in the presence of a religious holiday or a serious illness — sniffles don't count.

4. Collaboration Guidelines

You may *not* collaborate with your classmates on propositions (etc.) that have yet to be correctly presented in class. Yes, I know that this runs counter to the usual practice in the department, and yes, I believe that there are good reasons for this policy in this particular course (independence, the satisfaction of producing a proof that is *yours*, etc.). You may, however, talk to me.

Once the class agrees that a proposition has been correctly presented (or I assign an exercise for you to write up that will not be presented), however, you are free to collaborate with your peers on any write-ups that you do, subject to the following guidelines:

- You must indicate on your written work who your collaborators were (you may note different collaborators on different problems!)
- The actual write-up of your homework assignment should be done out of sight of any material produced during the collaboration (notebooks, scraps of paper, chalkboards, etc.) and without looking at any notes you may have taken in class. If you cannot write up the solution without wanting to refer to your collaboration material, then you probably have not yet understood the problem. In that case, you should delete anything you have produced so far on the problem, go back to your notes (etc.) and start again.

5. Exams

There will be an open-notes un-timed take-home final exam.

6. Grading

Your grade for the course is determined by your written work, your class presentations, and your performance on the final exam.[4] It is difficult to translate individual scores into an overall grade in a course structured like this one, but roughly speaking, doing the minimum required work (i.e. completing all assigned homework problems and the minimum number of presentations with decent scores) will yield a 3.0, while completing over twice the required minimum work, with excellent scores, will earn a 4.0. I will also take your trajectory into account, especially for the quality of the presentations, as I expect there to be some growing pains at the beginning of the class.

The three inputs into your final grade are weighted as follows:

Written Work: 50%
Class Presentations: 30%
Final: 20%

[4]I am deliberately *not* taking any qualitative measure of "class participation" into account, as my sense is that such grades are too easily subject to implicit bias.

7. Resources

7.1. **Office Hours.** Please stop by to see me during office hours! Ask questions! Hang out with your classmates! Learn how to draw cartoon animals!

7.2. **Math Question Center.** The MQC is place you can go to discuss homework with your classmates and ask questions of upperclass student tutors. The MQC is located in KINSC H011 (the same place as discussion sessions!) and is open Sunday through Thursday 7–9p.

7.3. **New Mathematicians' Study Center.** The New Mathematicians Study Center (NMSC) is a collaboration space for historically underrepresented or marginalized students in mathematics. The center is staffed by 1-2 undergraduates who are able to help students with some math courses. Students who identify as historically underrepresented or marginalized in mathematics are invited to stop by to work on homework, reinforce concepts learned previously in the course, or to simply meet with and get to know other math students. The NMSC meets 9–11p on Sundays in the OAR / STOKES 118K.

7.4. **Equal Access.** Haverford College is committed to providing equal access to students with a disability. If you have (or think you have) a learning difference or disability Ð including mental health, medical, or physical impairment, please contact the Office of Access and Disability Services (ADS) at `hc-ads@haverford.edu`. The Coordinator willconfidentially discuss the process to establish reasonable accommodations.

Students who have already been approved to receive academic accommodations and want to use their accommodations in this course should share their verification letter with me and also make arrangements to meet with me as soon as possible to discuss their the specific accommodations. Please note that accommodations are **not retroactive** and require advance notice to implement.

It is a state law in Pennsylvania that individuals must be given advance notice if they are to be recorded. Therefore, any student who has a disability-related need to audio record this class must first be approved for this accommodation from the Coordinator of Access and Disability Services and then must speak with me. Other class members will need to be aware that this class may be recorded.